Week 5 Homework – CMSC405

1. Show the results and intermediate steps for a translation of (20,40,-10), a rotation of 135 degrees about the z-axis applied to a starting point of (45,-95,20). Perform a separate operation applying scale with scale factors of sx=2.0, sy=1.6 and sz=1.0. You should use 4x4 matrix math for your calculations. Note: Use the $P_2 = T^{-1}$ R(z) T P1 approach for a general 3D rotation about the z-axis and $P_2 = T^{-1}$ S T P1 approach for a general scaling.

P1 = (45, -95, 20)

$$\mathrm{T} = \begin{pmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{T}^{-1} = \begin{pmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -40 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{R}(z) = \begin{pmatrix} cos(135) & -sin(135) & 0 & 0 \\ sin(135) & cos(135) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -0.7071 & -0.7071 & 0 & 0 \\ 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \mathrm{P2} &= \mathrm{T}^{-1} \times R(z) \times T \times P1 \\ &= \begin{pmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -40 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cos(135) & -sin(135) & 0 & 0 \\ sin(135) & cos(135) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 45 \\ -95 \\ 20 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -27.0711 \\ 44.8528 \\ 20 \\ 1 \end{pmatrix} \end{aligned}$$

$$\mathrm{S} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2.0 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \mathrm{P2} &= \mathrm{T}^{-1} \times S \times T \times P1 \\ &= \begin{pmatrix} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -40 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2.0 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 45 \\ -95 \\ 20 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -130 \\ -88 \\ 10 \\ 1 \end{pmatrix} \end{aligned}$$

2. Using quaternions, determine the final transformed location of point P1= (5,9,-10), after a 45-degree rotation about the z-axis, 90-degree rotation about the x-axis and 75-degree rotation about the y-axis. Be sure to show your work including the quaternion values for all steps.

P = (5, 9, -10) = 0 + 5i + 9j - 10k

q1 = cos(45/2) + i * sin(45/2) = 0.4518 + 0.8921i

q1' = 0.4518 - 0.8921k

P1 = q1 * P * q1' = (0.4518 + 0.8921k) (5i + 9j - 10k) (0.4518 - 0.8921k)

= (2.259i + 4.0662j - 4.518k + 4.4605ki + 8.0289kj - 8.921k^2) (0.4518 - 0.8921k)

= (2.259i + 4.0662j - 4.518k + 4.4605j - 8.0289i + 8.921) (0.4518 - 0.8921k)

= (8.921 -6.03i + 8.5312j - 4.518k) (0.4518 - 0.8921k)

= 4.0305 - 7.9584k - 2.724i + 5.3793ik + 3.8544j - 7.6107jk - 2.0412k + 4.0305k^2

= 4.0305 - 7.9584k - 2.724i - 5.3793j + 3.8544j - 7.6107i - 2.0412k - 4.0305

-10.3347i - 1.5249j - 9.9996k

q2 = cos(75/2) + j * sin(75/2) = 0.967 - 0.2548j

q2' = 0.967 + 0.2548j

P2 = q2 * P1 * q2' = (0.967 - 0.2548j) (-10.3347i - 1.5249j - 9.9996k) (0.967 + 0.2548j)

= (-9.9937i - 1.4746j - 9.67k + 2.6333ji + 0.3885j^2 + 2.5479jk) (0.967 + 0.2548j)

= (-9.9937i - 1.4746j - 9.67k - 2.6333k - 0.3885j + 2.5479i) (0.967 + 0.2548j)

= (-0.3885 - 7.4458i - 1.4746j - 12.303k) (0.967 + 0.2548j)

= -0.3757 - 0.099j - 7.2009i - 1.8466ij - 1.4259j - 0.3757j^2 - 11.8973k - 3.1349kj

= -0.3757 - 0.099j - 7.2009i - 1.8466k - 1.4259j + 0.3757 - 11.8973k + 3.1349i

= -4.066i - 1.5249j - 13.7439k

q3 = cos(90/2) + i * sin(90/2) = -0.5918 + 0.8061i

q3' = -0.5918 - 0.8061i

P3 = q3 * P2 * q3' = (-0.5918 + 0.8061i) (-4.066i - 1.5249j - 13.7439k) (-0.5918 - 0.8061i)

= (2.4063i + 0.9024j + 8.1336k - 3.2776i^2 - 12.292ij - 11.079ik) (-0.5918 - 0.8061i)

= (2.4063i + 0.9024j + 8.1336k + 3.2776 - 12.292k + 11.079j) (-0.5918 - 0.8061i)

= (3.2776 + 2.4063i + 11.9814j +6.9044k) (-0.5918 - 0.8061i)

= -1.9379 - 2.6421i - 1.2465i - 1.9397i^2 - 7.0906j - 9.6528ji - 4.086k - 5.5656ki

= -1.9379 - 2.6421i - 1.2465i + 1.9397 - 7.0906j + 9.6528k - 4.086k - 5.5656j

= -3.8886i -12.6562j + 5.5722k

P3 = (-3.8886, -12.6562, 5.5722)

3. Using OpenGL and your programming environment, create and provide 3D views for a 100 by 100 by 100 cube. You should use QL_QUADS to create each of the cube sides. Each side should be a different color (of your choice) and have text or a bitmap pattern of your choice. Your code should display each of the 6-sides using perspectives of your choice. However; each perspective should retain the 3D perspective. Hint: You can expand from the existing code example on pages 346-347 and build your cube one side at a time. No animation is required. You can provide the 6 perspectives by manually changing the parameters and submitting the snapshots in your document along with the parameter changes; or you can use C++ code to display each perspective in a loop. Either approach is acceptable.

Deliverables: You should submit a well-organized, word document that includes the results for the first two questions of this assignment along with a screen captures of the output of running your Visual C++ code. Be sure to include your parameters for the perspective changes and the snapshots from each of your 6 different perspectives. You should submit your C++ source code and header file for your 3D perspective applications. You should name your word assignment “yournamehw5.doc” (or .docx). You should name your C++ source code yournamehw5.cpp. Be sure to submit your homework in the WebTycho assignments folder no later than the due date listed in the syllabus.